A Motivational Introduction to Computational Logic and (Constraint) Logic Programming

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Computational Logic

- Logic of Computation
  - program verification
  - proving properties

- Declarative Programming
  - direct use of logic
  - as a programming tool

- Logic
- Algorithms
- Lambda calculus
- Logic and AI
- Knowledge representation
- Functional programming
- Logic programming
- Constraints
- Logic of programming
- Declarative programming
The Program Correctness Problem

- Conventional models of using computers – not easy to determine correctness!
  - Has become a very important issue, not just in safety-critical apps.
  - Components with assured quality, being able to give a warranty, ...
  - Being able to run untrusted code, certificate carrying code, ...
A Simple Imperative Program

- Example:

```c
#include <stdio.h>
main() {
    int Number, Square;
    Number = 0;
    while(Number <= 5)
        { Square = Number * Number;
          printf("%d\n",Square);
          Number = Number + 1; } }
```

- Is it correct? With respect to what?

- A suitable formalism:
  - to provide specifications (describe problems), and
  - to reason about the correctness of programs (their implementation).

is needed.
Natural Language

“Compute the squares of the natural numbers which are less or equal than 5.”

Ideal at first sight, but:

- verbose
- vague
- ambiguous
- needs context (assumed information)
- ...

Philosophers and Mathematicians already pointed this out a long time ago...
Logic

- A means of clarifying / formalizing the human thought process
- Logic for example tells us that (classical logic)
  - Aristotle likes cookies, and
  - Plato is a friend of anyone who likes cookies
  imply that
  - Plato is a friend of Aristotle
- Symbolic logic:
  A shorthand for classical logic – plus many useful results:
  \[ a_1 : \text{likes}(\text{aristotle, cookies}) \]
  \[ a_2 : \forall X \text{ likes}(X, \text{cookies}) \rightarrow \text{friend}(\text{plato, } X) \]
  \[ t_1 : \text{friend}(\text{plato, aristotle}) \]
  \[ T[a_1, a_2] \vdash t_1 \]
- But, can logic be used:
  - To represent the problem (specifications)?
  - Even perhaps to solve the problem?
Using Logic

- For expressing specifications and reasoning about the correctness of programs we need:
  - Specification languages (assertions), modeling, ...
  - Program semantics (models, axiomatic, fixpoint, ...).
  - Proofs: program *verification* (and debugging, equivalence, ...).
Generating Squares: A Specification (I)

Numbers — we will use “Peano” representation for simplicity:
0 → 0 1 → s(0) 2 → s(s(0)) 3 → s(s(s(0))) …

● Defining the natural numbers:
   \(\text{nat}(0) \land \text{nat}(s(0)) \land \text{nat}(s(s(0))) \land \ldots\)

● A better solution:
   \(\text{nat}(0) \land \forall X \ (\text{nat}(X) \rightarrow \text{nat}(s(X)))\)

● Order on the naturals:
   \(\forall X \ (\text{le}(0, X)) \land\)
   \(\forall X\forall Y \ (\text{le}(X, Y) \rightarrow \text{le}(s(X), s(Y)))\)

● Addition of naturals:
   \(\forall X \ (\text{nat}(X) \rightarrow \text{add}(0, X, X)) \land\)
   \(\forall X\forall Y\forall Z \ (\text{add}(X, Y, Z) \rightarrow \text{add}(s(X), Y, s(Z)))\)
Generating Squares: A Specification (II)

- **Multiplication of naturals:**
  \[
  \forall X \ (\text{nat}(X) \rightarrow \text{mult}(0, X, 0)) \land \\
  \forall X \forall Y \forall Z \forall W \ (\text{mult}(X, Y, W) \land \text{add}(W, Y, Z) \rightarrow \text{mult}(s(X), Y, Z))
  \]

- **Squares of the naturals:**
  \[
  \forall X \forall Y \ (\text{nat}(X) \land \text{nat}(Y) \land \text{mult}(X, X, Y) \rightarrow \text{nat\_square}(X, Y))
  \]

We can now write a *specification* of the (imperative) program, i.e., conditions that we want the program to meet:

- **Precondition:**
  empty.

- **Postcondition:**
  \[
  \forall X (\text{output}(X) \leftarrow (\exists Y \ \text{nat}(Y) \land \text{le}(Y, s(s(s(s(0)))))) \land \text{nat\_square}(Y, X))
  \]
Alternative Use of Logic?

- So, logic allows us to *represent problems* (program specifications).

  But, it would be interesting to also improve:

  i.e., the process of implementing solutions to problems.

  - The importance of Programming Languages (and tools).
  - Interesting question: can logic help here too?
From Representation/Specification to Computation

- Assuming the existence of a *mechanical proof method* (deduction procedure)
  a *new view of problem solving and computing is possible* [Greene]:
  - program once and for all the deduction procedure in the computer,
  - find a suitable *representation* for the problem (i.e., the *specification*),
  - then, to obtain solutions, ask questions and let deduction procedure do rest:

\[\text{Problem} \rightarrow \text{Questions} \rightarrow \text{Deduction system} \rightarrow \text{(Correct) Answers / Results}\]

- No correctness proofs needed!
## Computing With Our Previous Description / Specification

<table>
<thead>
<tr>
<th>Query</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$nat(s(0))$ ?</td>
<td>$(yes)$</td>
</tr>
<tr>
<td>$\exists X\ add(s(0), s(s(0)), X)$ ?</td>
<td>$X = s(s(s(0)))$</td>
</tr>
<tr>
<td>$\exists X\ add(s(0), X, s(s(s(0))))$ ?</td>
<td>$X = s(s(0))$</td>
</tr>
<tr>
<td>$\exists X\ nat(X)$ ?</td>
<td>$X = 0 \lor X = s(0) \lor X = s(s(0)) \lor \ldots$</td>
</tr>
<tr>
<td>$\exists X\exists Y\ add(X, Y, s(0))$ ?</td>
<td>$(X = 0 \land Y = s(0)) \lor (X = s(0) \land Y = 0)$</td>
</tr>
<tr>
<td>$\exists X\ nat_square(s(s(0)), X)$ ?</td>
<td>$X = s(s(s(0))))$</td>
</tr>
<tr>
<td>$\exists X\ nat_square(X, s(s(s(s(0))))))$ ?</td>
<td>$X = s(s(0))$</td>
</tr>
<tr>
<td>$\exists X\exists Y\ nat_square(X, Y)$ ?</td>
<td>$(X = 0 \land Y = 0) \lor (X = s(0) \land Y = s(0)) \lor (X = s(s(0)) \land Y = s(s(s(s(0)))))) \lor \ldots$</td>
</tr>
<tr>
<td>$\exists X\ output(X)$ ?</td>
<td>$X = 0 \lor X = s(0) \lor X = s(s(s(s(0)))) \lor X = s^9(0) \lor X = s^{16}(0) \lor X = s^{25}(0)$</td>
</tr>
</tbody>
</table>
Which Logic?

- We have already argued the convenience of representing the problem in logic, but
  - which logic?
    * propositional
    * predicate calculus (first order)
    * higher-order logics
    * modal logics
    * $\lambda$-calculus, ...
  - which reasoning procedure?
    * natural deduction, classical methods
    * resolution
    * Prawitz/Bibel, tableaux
    * bottom-up fixpoint
    * rewriting
    * narrowing, ...
Issues

- We try to maximize expressive power.
- But one of the main issues is whether we have an **effective** reasoning procedure.
- It is important to understand the underlying properties and the theoretical limits!
- Example: propositions vs. first-order formulas.
  - **Propositional logic:**
    - “spot is a dog” \( p \)
    - “dogs have tail” \( q \)
    - but how can we conclude that Spot has a tail?
  - **Predicate logic extends the expressive power of propositional logic:**
    - \( \text{dog}(\text{spot}) \)
    - \( \forall X \text{dog}(X) \rightarrow \text{has\_tail}(X) \)
    - now, using deduction we can conclude:
      - \( \text{has\_tail}(\text{spot}) \)
Comparison of Logics (I)

- Propositional logic:
  
  "spot is a dog" \(\ p\)
  
  + decidability/completeness
  
  - limited expressive power
  
  + practical deduction mechanism
  
  \(\rightarrow\) circuit design, “answer set” programming, ...

- Predicate logic: (first order)

  "spot is a dog" \(\text{dog}(\text{spot})\)

  +/- decidability/completeness

  +/- good expressive power

  + practical deduction mechanism (e.g., SLD-resolution)

  \(\rightarrow\) classical logic programming!
Comparison of Logics (II)

- Higher-order predicate logic:
  - “There is a relationship for spot” \( X(spot) \)
  - decidability/completeness
  - good expressive power
  - practical deduction mechanism

But interesting subsets \( \rightarrow \) HO logic programming, functional-logic prog., ...

- Other logics: decidability? Expressive power? Practical deduction mechanism?
  Often (very useful) variants of previous ones:
    - Predicate logic + constraints (in place of unification)
      \( \rightarrow \) constraint programming!
    - Propositional temporal logic, etc.

- Interesting case: \( \lambda \)-calculus
  - similar to predicate logic in results, allows higher order
  - does not support predicates (relations), only functions

\( \rightarrow \) functional programming!
Generating squares by SLD-Resolution – Logic Programming (I)

- We code the problem as definite (Horn) clauses:
  
  \[\begin{align*}
  & \text{nat}(0) \\
  & \neg \text{nat}(X) \lor \text{nat}(\text{s}(X)) \\
  & \neg \text{nat}(X) \lor \text{add}(0, X, X)) \\
  & \neg \text{add}(X, Y, Z) \lor \text{add}(\text{s}(X), Y, \text{s}(Z)) \\
  & \neg \text{nat}(X) \lor \text{mult}(0, X, 0) \\
  & \neg \text{mult}(X, Y, W) \lor \neg \text{add}(W, Y, Z) \lor \text{mult}(\text{s}(X), Y, Z) \\
  & \neg \text{nat}(X) \lor \neg \text{nat}(Y) \lor \neg \text{mult}(X, X, Y) \lor \text{nat} \_ \text{square}(X, Y)
  \end{align*}\]

- **Query:** \( \text{nat}(\text{s}(0)) \) ?

- In order to refute: \( \neg \text{nat}(\text{s}(0)) \)

- Resolution:
  
  \( \neg \text{nat}(\text{s}(0)) \) with \( \neg \text{nat}(X) \lor \text{nat}(\text{s}(X)) \) gives \( \neg \text{nat}(0) \)
  
  \( \neg \text{nat}(0) \) with \( \text{nat}(0) \) gives \( \square \)

- **Answer:** \( \text{yes} \)
Generating squares by SLD-Resolution – Logic Programming (II)

\[
\begin{align*}
nat(0) \\
\neg nat(X) \lor nat(s(X)) \\
\neg nat(X) \lor add(0, X, X) \\
\neg add(X, Y, Z) \lor add(s(X), Y, s(Z)) \\
\neg nat(X) \lor mult(0, X, 0) \\
\neg mult(X, Y, W) \lor \neg add(W, Y, Z) \lor mult(s(X), Y, Z) \\
\neg nat(X) \lor \neg nat(Y) \lor \neg mult(X, X, Y) \lor nat\_square(X, Y)
\end{align*}
\]

- **Query:** \( \exists X \exists Y \ add(X, Y, s(0)) \quad ? \)
- **In order to refute:** \( \neg add(X, Y, s(0)) \)
- **Resolution:**
  \( \neg add(X, Y, s(0)) \) with \( \neg nat(X) \lor add(0, X, X) \) gives \( \neg nat(s(0)) \)
  \( \neg nat(s(0)) \) solved as before
- **Answer:** \( X = 0, Y = s(0) \)
- **Alternative:**
  \( \neg add(X, Y, s(0)) \) with \( \neg add(X, Y, Z) \lor add(s(X), Y, s(Z)) \) gives \( \neg add(X, Y, 0) \)
Generating Squares in a Practical Logic Programming System (I)

:- module(_,_,[’bf/af’]).

nat(0) <- .
nat(s(X)) <- nat(X).

le(0,_X) <- .
le(s(X),s(Y)) <- le(X,Y).

add(0,Y,Y) <- nat(Y).
add(s(X),Y,s(Z)) <- add(X,Y,Z).

mult(0,Y,0) <- nat(Y).
mult(s(X),Y,Z) <- add(W,Y,Z), mult(X,Y,W).

nat_square(X,Y) <- nat(X), nat(Y), mult(X,X,Y).

output(X) <- nat(Y), le(Y,s(s(s(s(s(0)))))), nat_square(Y,X).
## Generating Squares in a Practical Logic Programming System (II)

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<tr>
<td>?- nat(s(0)).</td>
<td>yes</td>
</tr>
<tr>
<td>?- add(s(0),s(s(0)),X).</td>
<td>X = s(s(s(0)))</td>
</tr>
<tr>
<td>?- add(s(0),X,s(s(s(0)))).</td>
<td>X = s(s(0))</td>
</tr>
<tr>
<td>?- nat(X).</td>
<td>X = 0 ; X = s(0) ; X = s(s(0)) ; ...</td>
</tr>
<tr>
<td>?- add(X,Y,s(0)).</td>
<td>(X = 0 , Y=s(0)) ; (X = s(0) , Y = 0)</td>
</tr>
<tr>
<td>?- nat_square(s(s(0)), X).</td>
<td>X = s(s(s(s(0))))</td>
</tr>
<tr>
<td>?- nat_square(X,s(s(s(s(0)))))</td>
<td>X = s(s(0))</td>
</tr>
<tr>
<td>?- nat_square(X,Y).</td>
<td>(X = 0 , Y=0) ; (X = s(0) , Y=s(0)) ; (X = s(s(0)) , Y=s(s(s(s(0)))))) ; ...</td>
</tr>
<tr>
<td>?- output(X).</td>
<td>X = 0 ; X = s(0) ; X = s(s(s(s(0)))) ; ...</td>
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</table>
**Introductory example (I) – Family relations**

\[
\begin{align*}
\text{father_of}(\text{john}, \text{peter}) \\
\text{father_of}(\text{john}, \text{mary}) \\
\text{father_of}(\text{peter}, \text{michael}) \\
\text{mother_of}(\text{mary}, \text{david}) \\
\forall X \forall Y (\exists Z (\text{father_of}(X, Z) \land \text{father_of}(Z, Y)) \rightarrow \text{grandfather_of}(X, Y)) \\
\forall X \forall Y (\exists Z (\text{father_of}(X, Z) \land \text{mother_of}(Z, Y)) \rightarrow \text{grandfather_of}(X, Y))
\end{align*}
\]

\[
\begin{align*}
\text{father_of}(\text{john}, \text{peter}). \\
\text{father_of}(\text{john}, \text{mary}). \\
\text{father_of}(\text{peter}, \text{michael}). \\
\text{mother_of}(\text{mary}, \text{david}).
\end{align*}
\]

\[
\text{grandfather_of}(L, M) :- \text{father_of}(L, K), \\
\text{father_of}(K, M).
\]

\[
\text{grandfather_of}(X, Y) :- \text{father_of}(X, Z), \\
\text{mother_of}(Z, Y).
\]

- How can \text{grandmother_of}/2 be represented?
- What does \text{grandfather_of}(X, \text{david}) mean? And \text{grandfather_of}(\text{john}, X)?
A (very brief) History of Logic Programming (I)

- **60’s**
  - Greene: problem solving.
  - Robinson: linear resolution.

- **70’s**
  - *(early)* Kowalski: procedural interpretation of Horn clause logic. Read: 
    \[ A \text{ if } B_1 \text{ and } B_2 \text{ and } \cdots \text{ and } B_n \text{ as:} \]
    to solve (execute) \( A \), solve (execute) \( B_1 \) and \( B_2 \) and,..., \( B_n \)
  - *(early)* Colmerauer: specialized theorem prover (Fortran) embedding the procedural interpretation: Prolog (Programmation et Logique).
  - In the U.S.: “next-generation AI languages” of the time (i.e. planner) seen as inefficient and difficult to control.
**A (very brief) History of Logic Programming (II)**

- **Late 80’s, 90’s**
  - Major research in the basic paradigms and advanced implementation techniques: Japan (Fifth Generation Project), US (MCC), Europe (ECRC, ESPRIT projects).
  - Numerous commercial Prolog implementations, programming books, and a *de facto* standard, the Edinburgh Prolog family.
  - First parallel and concurrent logic programming systems.
  - CLP – Constraint Logic Programming: Major extension – many new applications areas.
Currently

- Many commercial CLP systems with fielded applications.
- Extensions to full higher order, inclusion of functional programming, ...
- Highly optimizing compilers, automatic parallelism, automatic debugging.
- Concurrent constraint programming systems.
- Distributed systems.
- Object oriented dialects.
- Applications
  - Natural language processing
  - Scheduling/Optimization problems
  - AI related problems
  - (Multi) agent systems programming.
  - Program analyzers
  - ...

...